A mechanism for the latent liquidity revealing into the limit order book

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Master degree in Physics of complex systems

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Nomenclature

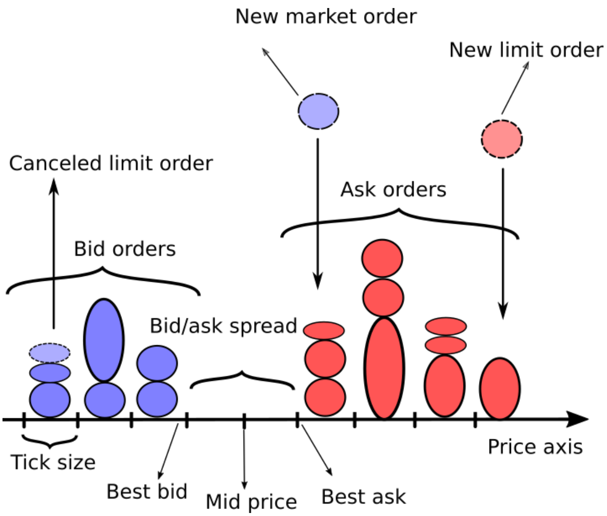

Figure: A sketch of the limit order book with the main quantities

Few observations

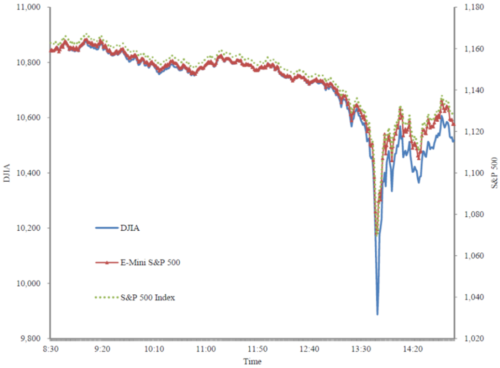

Figure: The flash crash of May 6, 2010
Source: [KKST11]

- Prices fluctuations are only weakly dependent on the news feed [JLGB08]
- Most of daily volatility is caused by market activity
- The instantaneous liquidity is approximately 0.1% of the total daily traded volume [TPLF15]

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Presentation of the model in the infinite memory limit

Refer to [DBMB15]

$$
\begin{aligned}
\partial_t \rho_{\mathrm{A}}(x,t) &= D\partial_{xx}\rho_{\mathrm{A}}(x,t) - R_{A,B} \\
\partial_t \rho_{\mathrm{B}}(x,t) &= D\partial_{xx}\rho_{\mathrm{B}}(x,t) - R_{A,B}
\end{aligned}
$$

It predicts the square root law of the impact:

$$
\mathcal{I}(\mathcal{Q}_t) \propto \sqrt{\mathcal{Q}_t} \tag{2}
$$

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Presentation of the model

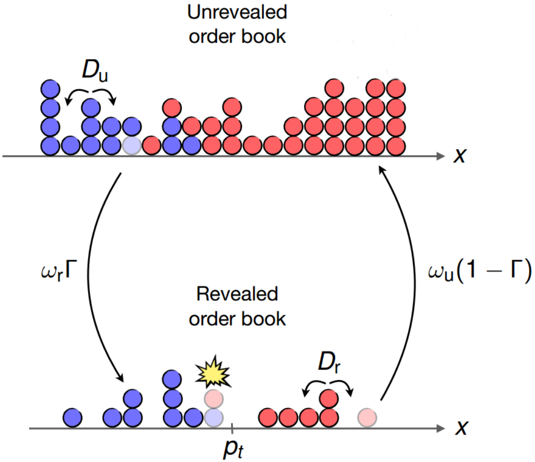

Figure: A sketch of the model

The equations

Ask side

$$\partial_t \rho_A^{(r)} = D_r \partial_{xx} \rho_{A^{(r)}} + \omega_r \Gamma(k(x - p_t)) \rho_A^{(u)} - \omega_u \left[1 - \Gamma(k(x - p_t))\right] \rho_A^{(r)} - \kappa \rho_A^{(r)} \rho_B^{(r)}$$

$$\partial_t \rho_A^{(u)} = D_u \partial_{xx} \rho_A^{(u)} - \omega_r \Gamma(k(x - p_t)) \rho_A^{(u)} + \omega_u \left[1 - \Gamma(k(x - p_t))\right] \rho_A^{(r)}$$

Bid side

$$\partial_t \rho_B^{(r)} = D_r \partial_{xx} \rho_B^{(r)} + \omega_r \Gamma(k(p_t - x)) \rho_B^{(u)} - \omega_u \left[1 - \Gamma(k(p_t - x))\right] \rho_B^{(r)} - \kappa \rho_A^{(r)} \rho_B^{(r)}$$

$$\partial_t \rho_B^{(u)} = D_u \partial_{xx} \rho_B^{(u)} - \omega_r \Gamma(k(p_t - x)) \rho_B^{(u)} + \omega_u \left[1 - \Gamma(k(p_t - x))\right] \rho_B^{(r)}$$

We define

$$\begin{aligned} \phi_{\mathrm{r}} &:= \rho_{\mathrm{B}}^{(\mathrm{r})} - \rho_{\mathrm{A}}^{(\mathrm{r})} \\ \ell_{\mathrm{u/r}} &:= \sqrt{\frac{D_{\mathrm{u/r}}}{\omega}} \\ \Gamma(y) &= \begin{cases} e^{-y}, & y \geq 0 \\ 1, & y < 0 \end{cases} \\ \xi_t &:= x - p_t \end{aligned}$$

Stationary solutions

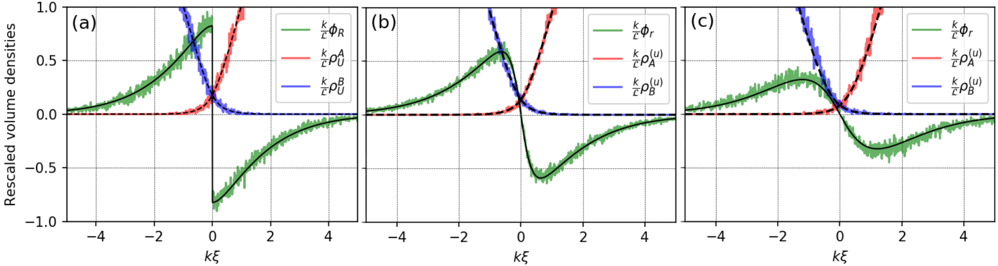

Figure: Stationary analytical solution of the book compared to the numerical simulation for (a) $\ell_r = 0$, $k\ell_u = 0.375$ (b) $\ell_r \neq \ell_u, k\ell_u = 0.375$, $\ell_u/\ell_r = 3.214$ (c) $\ell_r = \ell_u$, $k\ell_u = 0.375$

Stability

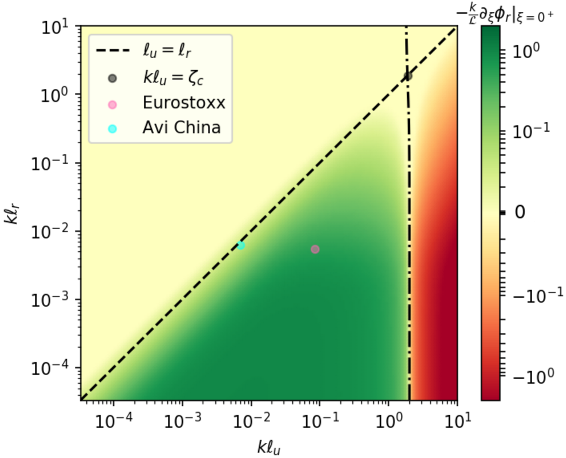

Figure: Study of the stability of the system

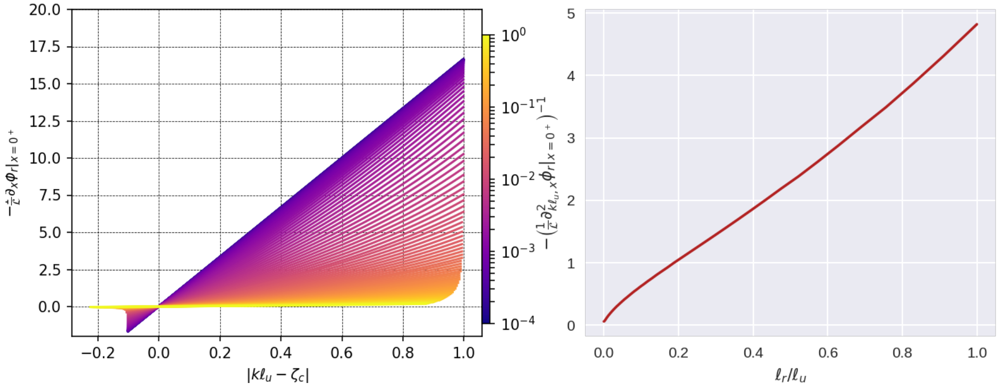

Figure: Behavior of the slope at the transition

We define the fair price p_t^f as:

$$\int_0^{p_t^f} d\xi \left[\rho_{\mathrm{A}}^{(\mathrm{u})}(\xi,t)+\rho_{\mathrm{A}}^{(\mathrm{r})}(\xi,t)\right]=\int_{p_t^f}^{\infty} d\xi \left[\rho_{\mathrm{B}}^{(\mathrm{u})}(\xi,t)+\rho_{\mathrm{B}}^{(\mathrm{r})}(\xi,t)\right]$$

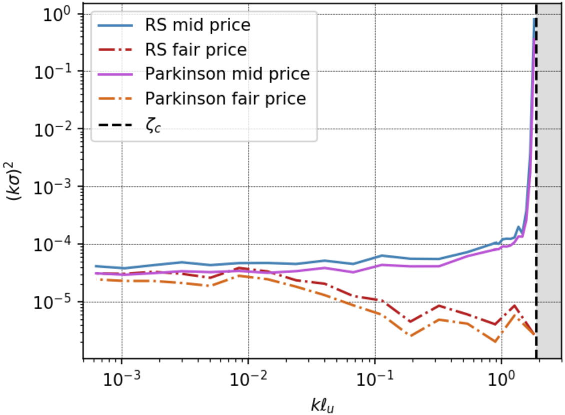

Figure: Numerical study of the volatility for $\ell_{\mathrm{r}} = \ell_{\mathrm{u}}$ as a function of $k\ell_{\mathrm{u}}$

Recap

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- The level of liquidity is mainly determined by the value of of $k\ell_{\mathrm{u}}$
- When approaching the critical condition the liquidity diminishes and the volatility consequently increases
- As the value of $k\ell_{\mathrm{r}}$ diminishes the liquidity is concentrated around the origin and the fluctuations are consequently smaller
- The critical condition happens when the overlap is of order of the market depth

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$D_{\mathbf{r}} = D_{\mathbf{u}}$

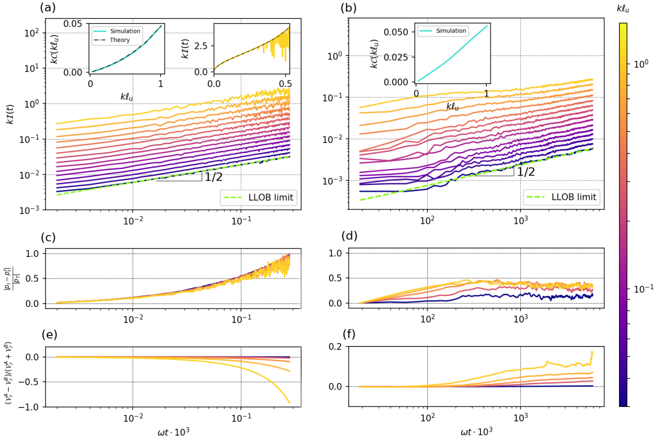

Figure: (a) Fast execution rate, (b) Slow execution rate

$D_r = 0$

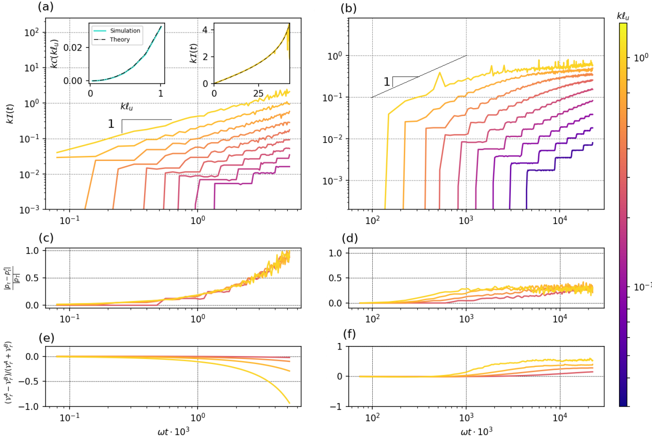

Figure: (a) Fast execution rate, (b) Slow execution rate

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The model we here presented:

- Is the first to our knowledge to connect latent liquidity to the real limit order book
- Predicts the average shape of the limit order book and it is in good agreement with real data, allowing to fit some parameters of the model
- Predicts square root impact
- Allows promising extensions to a multi-time scale version of it

Jonathan Donier, Julius Bonart, Iacopo Mastromatteo, and J-P Bouchaud.
A fully consistent, minimal model for non-linear market impact.
Quantitative finance, 15(7):1109–1121, 2015.

Armand Joulin, Augustin Lefevre, Daniel Grunberg, and Jean-Philippe Bouchaud.
Stock price jumps: news and volume play a minor role.
arXiv preprint arXiv:0803.1769, 2008.

Andrei Kirilenko, Albert S Kyle, Mehrdad Samadi, and Tugkan Tuzun.
The flash crash: The impact of high frequency trading on an electronic market.
Available at SSRN, 1686004, 2011.

Bence Toth, Imon Palit, Fabrizio Lillo, and J Doyne Farmer.
Why is equity order flow so persistent?
Journal of Economic Dynamics and Control, 51:218–239, 2015.

Thank you!